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Pomeron–Odderon interference effects in electroproduction of two pions

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Abstract. We study Pomeron–Odderon interference effects giving rise to charge and single-spin asymmetries in diffractive electroproduction of a $\pi^+\pi^-$ pair. We calculate these asymmetries, originating from both longitudinal and transverse polarizations of the virtual photon, in the framework of QCD and in the Born approximation, in a kinematical domain accessible to HERA experiments. We predict a sizable charge asymmetry with a characteristic dependence on the invariant mass of the $\pi^+\pi^-$ pair, which makes this observable very important for establishing the magnitude of the Odderon exchange in hard processes. The single-spin asymmetry turns out to be rather small. We briefly discuss future improvements of our calculations and their possible effects on the results.

1 Introduction

Hadronic reactions at low momentum transfer and high energies are described in the framework of QCD in terms of the dominance of color singlet exchanges corresponding to a few reggeized gluons. The charge conjugation-even sector of the t-channel exchanges is understood as the QCD-Pomeron described by the BFKL equation [1]. The charge-odd exchange is less well understood, although the corresponding BKP equations [2] have attracted much attention recently [3–6], thus reviving the relevance of phenomenological studies of the Odderon exchange pointed out years ago in [7]. Recent studies [8] confirm indeed the need for the Odderon contribution, in particular the need to understand the different behaviors of pp and $\bar{p}p$ elastic cross sections in the dip region. However studies of specific channels where the Odderon contribution is expected to be singled out have turned out to be very disappointing. Recent experimental studies at HERA of exclusive π^0 photoproduction [9] indicate a very small cross section for this process which stays in contradiction with theoretical predictions based on the stochastic vacuum model [10]. In diffractive η_c-meson photoproduction, the QCD prediction for the cross section is rather small [11,12] at the Born level; the inclusion of the evolution following from the BKP equation [13] leads to an increase of the predicted cross section for this process by one order of magnitude but no experimental data exist so far.

A new strategy to reveal the features of the charge-odd exchange is thus required. For that purpose let us first note that in all above mentioned meson production processes the scattering amplitude describing Odderon exchange enters quadratically in the cross section. This observation leads to the suggestion in [14] that the study of observables where Odderon effects are present at the amplitude level – and not at the squared amplitude level – is mandatory to get a convenient sensitivity to a rather small normalization of this contribution. This may be achieved by means of charge asymmetries, as for instance in open charm production [14]. Since the final state quark–antiquark pair has no definite charge parity both Pomeron and Odderon exchanges contribute to this process. Another example [15] is the charge asymmetry in soft photoproduction of two pions. On the other hand, the difficulty with the understanding of soft processes in QCD calls for studies of Odderon contributions in hard processes, such as electroproduction, where factorization properties allow for a perturbative calculation of the short-distance part of the scattering amplitude.

In a recent paper [16] we proposed to study the diffractive electroproduction of a $\pi^+\pi^-$ pair to search for the QCD-Odderon at the amplitude level. The $\pi^+\pi^-$ state does not have any definite charge parity and therefore both Pomeron and Odderon exchanges contribute. The originality of our study of the electroproduction process with respect to [14,15] is that we work in a perturbative QCD framework which enables us to derive more founded predictions in an accessible kinematical domain.

In this paper, we study in full detail the charge and single-spin asymmetries in the deeply virtual production of two pions within perturbative QCD, see Fig. 1. The ap-

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plication of pQCD for the calculation of a part of this process is justified by the presence of a hard scale: the squared mass $-Q^2$ of the virtual photon, Q^2 being of the order of a few GeV^2. The amplitude of this process includes the convolution of a perturbatively calculable hard subprocess with two non-perturbative inputs, the two pion generalized distribution amplitude (GDA) and the Pomeron–Odderon (P/O) proton impact factors. Since the $\pi^+\pi^-$ system is not a charge parity eigenstate, the GDA includes two charge parity components and allows for a study of the corresponding interference term. The relevant GDA is here just given by the light cone wave function of the two pion system [17].

In this paper we supplement our previous work [16] by the inclusion of contributions from transversely polarized photons to the charge asymmetry. Additionally we study the single-spin asymmetry which is proportional to the interference of non-diagonal, i.e. longitudinal-transverse polarization terms. In contrast the charge asymmetry picks up contributions of all possible polarization combinations, which, as expected, turn out to be of the same order of magnitude at moderate Q^2.

Since transversely polarized pion pairs are the only source of the dependence of the amplitude on the azimuthal angle of the pions in their c.m. frame, the amplitudes and cross sections are independent of this angle in our approximation. As a result, the transverse charge asymmetry [15], resulting from a distribution in this angle, is zero.

Our results for the charge and spin asymmetries, which have been obtained by a lowest order calculation, can be extended by the inclusion of evolution from the BFKL and BKP equations, in a similar way as it has been done in [13].

2 Kinematics

Let us first specify the kinematics of the process under study, namely the electron–proton scattering

$$e(p_i,\lambda)N(p_N) \to e(p_f)\pi^+(p_+)\pi^-(p_-)N'(p_{N'}), \qquad (1)$$

which proceeds through a virtual photon–proton reaction (Fig. 1)

$$\gamma^*(q,\epsilon)N(p_N) \to \pi^+(p_+)\pi^-(p_-)N'(p_{N'}), \qquad (2)$$

where λ is the initial electron helicity and ϵ the virtual photon polarization vector.

We introduce a Sudakov representation of all particle momenta using the Sudakov light-like momenta p_1, p_2. The virtual photon momentum can then be written as

$$q^\mu = p_1^\mu - \frac{Q^2}{s} p_2^\mu, \qquad (3)$$

where $s = 2p_1 \cdot p_2$. Similarly, the nucleon momentum in the initial state can be expressed through

$$p_N^\mu = p_2^\mu + \frac{M^2}{s} p_1^\mu, \qquad (4)$$

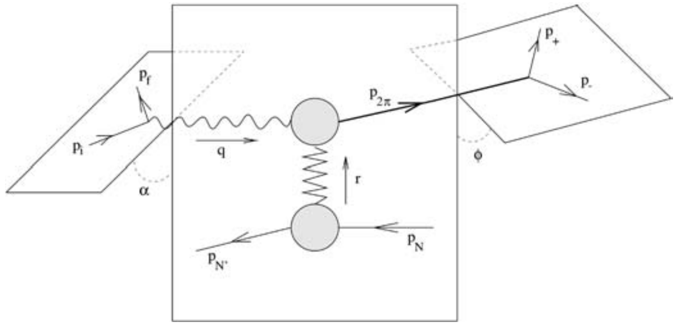

Fig. 1. Kinematics of the electroproduction of two pions

where M is the proton target mass. The variable s is related to the total energy squared of the virtual photon–proton system by

$$(q + p_N)^2 \approx s - Q^2 + M^2 \approx s.$$

As usual, y is the energy fraction carried by the virtual photon

$$y = \frac{q \cdot p_2}{p_i \cdot p_2}. \qquad (5)$$

The momentum of the two pion system is given by

$$p_{2\pi}^\mu = \left(1 - \frac{\vec{p}_{2\pi}^{\,2}}{s}\right) p_1^\mu + \frac{m_{2\pi}^2 + \vec{p}_{2\pi}^{\,2}}{s} p_2^\mu + p_{2\pi\perp}^\mu,$$
$$p_{2\pi\perp}^2 = -\vec{p}_{2\pi}^{\,2}. \qquad (6)$$

We denote by α the angle between the euclidean vectors $\vec{p}_i$ and $\vec{p}_{2\pi}$.

The quark momentum l_1 and antiquark momentum l_2 inside the loop before the formation of the two pion system (see Fig. 2) are parameterized as

$$l_1^\mu = z p_1^\mu + \frac{m^2 + (\vec{l} + z\vec{p}_{2\pi})^2}{zs} p_2^\mu + (l_\perp + z p_{2\pi\perp})^\mu, \qquad (7)$$

$$l_2^\mu = \bar{z} p_1^\mu + \frac{m^2 + (-\vec{l} + \bar{z}\vec{p}_{2\pi})^2}{\bar{z}s} p_2^\mu + (-l_\perp + \bar{z} p_{2\pi\perp})^\mu, \qquad (8)$$

where $2\vec{l}$ is the relative transverse momentum of the quarks forming the two pion system and $\bar{z} = 1 - z$, up to small corrections of the order $\vec{p}_{2\pi}^{\,2}/s$. Following the collinear approximation of the factorization procedure in the description of the two pion formation through the generalized distribution amplitude we put $\vec{l} = \vec{0}$ in the hard amplitude.

In a similar way as in (7) and (8) we parameterize the momenta of the produced pions as

$$p_+^\mu = \zeta p_1^\mu + \frac{m_\pi^2 + (\vec{p} + \zeta\vec{p}_{2\pi})^2}{\zeta s} p_2^\mu + (p_\perp + \zeta p_{2\pi\perp})^\mu, \qquad (9)$$

$$p_-^\mu = \bar{\zeta} p_1^\mu + \frac{m_\pi^2 + (-\vec{p} + \bar{\zeta}\vec{p}_{2\pi})^2}{\bar{\zeta} s} p_2^\mu + (-p_\perp + \bar{\zeta} p_{2\pi\perp})^\mu, \qquad (10)$$

where $2\vec{p}$ is now their relative transverse momentum, $\zeta = p_2 \cdot p_+ / p_2 \cdot p_{2\pi}$ is the fraction of the longitudinal momentum $p_{2\pi}$ carried by the produced π^+, and $\bar{\zeta} = 1 - \zeta$. The

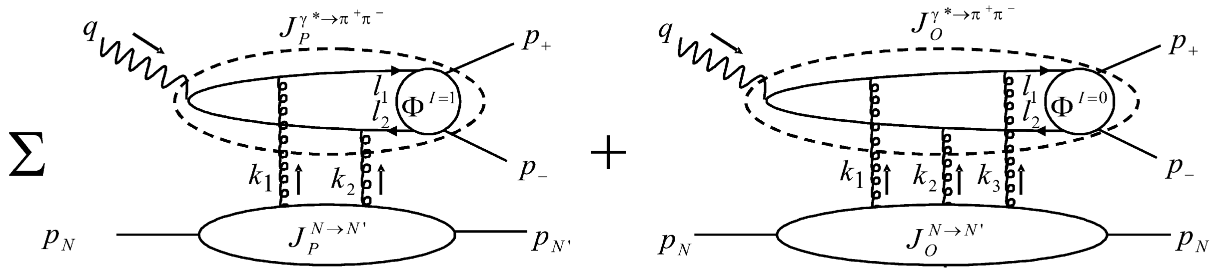

Fig. 2. Feynman diagrams describing $\pi^+\pi^-$ electroproduction in the Born approximation

variable ζ is related to the polar decay angle θ which is in the rest frame of the pion pair defined by

$$\beta\cos\theta = 2\zeta - 1, \quad \beta \equiv \sqrt{1 - \frac{4m_\pi^2}{m_{2\pi}^2}}. \tag{11}$$

Since the "longitudinal part" of the two pion wave function depends only on the angle θ and does not depend on the azimuthal decay angle ϕ (in the same rest frame of the pair) we focus on the calculation of forward–backward asymmetries expressed in terms of θ (see below). The squared momentum transfer $t = r^2$ ($r^\mu = p_{2\pi}^\mu - q^\mu$) can be written as

$$t = r^2 = -\vec{p}_{2\pi}^{\,2} + t_{\min}, \quad t_{\min} = -\frac{M^2(Q^2 + m_{2\pi}^2)^2}{s^2}. \tag{12}$$

3 Scattering amplitudes

It is well known (see e.g. [18,12] and references therein) that for large values of s, large Q^2 and small momentum transfer t the scattering amplitudes can be represented as convolutions over the two-dimensional transverse momenta of the t-channel gluons.

For the Pomeron exchange, which corresponds in the Born approximation of QCD to the exchange of two gluons in a color singlet state, see Fig. 2, the impact representation has the form

$$\mathcal{M}_P = -\mathrm{i}s \int \frac{\mathrm{d}^2\vec{k}_1 \mathrm{d}^2\vec{k}_2 \delta^{(2)}(\vec{k}_1 + \vec{k}_2 - \vec{p}_{2\pi})}{(2\pi)^2 \vec{k}_1^2 \vec{k}_2^2} \times J_P^{\gamma^*\to\pi^+\pi^-}(\vec{k}_1, \vec{k}_2) \cdot J_P^{N\to N'}(\vec{k}_1, \vec{k}_2), \tag{13}$$

where $J_P^{\gamma^*\to\pi^+\pi^-}(\vec{k}_1, \vec{k}_2)$ and $J_P^{N\to N'}(\vec{k}_1, \vec{k}_2)$ are the impact factors for the transition $\gamma^* \to \pi^+\pi^-$ via Pomeron exchange and of the nucleon in the initial state N into the nucleon in the final state N'.

The corresponding representation for the Odderon exchange, i.e. the exchange of three gluons in a color singlet state, is given by the formula

$$\mathcal{M}_O = -\frac{8\pi^2 s}{3!} \int \frac{\mathrm{d}^2\vec{k}_1 \mathrm{d}^2\vec{k}_2 \mathrm{d}^2\vec{k}_3 \delta^{(2)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_{2\pi})}{(2\pi)^6 \vec{k}_1^2 \vec{k}_2^2 \vec{k}_3^2} \times J_O^{\gamma^*\to\pi^+\pi^-} \cdot J_O^{N\to N'}, \tag{14}$$

where $J_O^{\gamma^*\to\pi^+\pi^-}(\vec{k}_1, \vec{k}_2, \vec{k}_3)$ and $J_O^{N\to N'}(\vec{k}_1, \vec{k}_2, \vec{k}_2)$ are the impact factors for the transition $\gamma^* \to \pi^+\pi^-$ via Odderon exchange and of the nucleon in initial state N into the nucleon in the final state N'.

The upper impact factors are calculated by the use of standard methods, see e.g. [19] and references therein.

3.1 Impact factors for $\gamma^*_{\mathbf{L}/\mathbf{T}} \to \pi^+\pi^-$

The leading order calculation in pQCD of the upper impact factors gives in the case of a longitudinal polarized photon

$$J_P^{\gamma_L^*}(\vec{k}_1, \vec{k}_2) = -\frac{\mathrm{i}eg^2\delta^{ab}Q}{2N_C} \int_0^1 \mathrm{d}z z\bar{z} P_P(\vec{k}_1, \vec{k}_2) \Phi^{I=1}(z, \zeta, m_{2\pi}^2), \tag{15}$$

where $\vec{k}_1 + \vec{k}_2 = \vec{p}_{2\pi}$ and the function $P_P(\vec{k}_1, \vec{k}_2)$ is given by

$$P_P(\vec{k}_1, \vec{k}_2) = \frac{1}{z^2\vec{p}_{2\pi}^{\,2} + \mu^2} + \frac{1}{\bar{z}^2\vec{p}_{2\pi}^{\,2} + \mu^2} - \frac{1}{(\vec{k}_1 - z\vec{p}_{2\pi})^2 + \mu^2} - \frac{1}{(\vec{k}_1 - \bar{z}\vec{p}_{2\pi})^2 + \mu^2}, \tag{16}$$

with $\mu^2 = m_q^2 + z\bar{z}Q^2$, where m_q is the quark mass and we put $m_u \simeq m_d = 0.006\,\mathrm{GeV}$. The GDA $\Phi^{I=0,1}(z, \zeta, m_{2\pi}^2)$ for isospin $I = 0, 1$ will be discussed in detail in the next section. The computation of the three-gluon exchange graphs for the longitudinally polarized photon results in the following impact factor:

$$J_O^{\gamma_L^*}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = -\frac{\mathrm{i}eg^3d^{abc}Q}{4N_C} \int_0^1 \mathrm{d}z z\bar{z} P_O(\vec{k}_1, \vec{k}_2, \vec{k}_3) \times \frac{1}{3}\Phi^{I=0}(z, \zeta, m_{2\pi}^2), \tag{17}$$

where $\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = \vec{p}_{2\pi}$ and

$$P_O(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{1}{z^2\vec{p}_{2\pi}^{\,2} + \mu^2} - \frac{1}{\bar{z}^2\vec{p}_{2\pi}^{\,2} + \mu^2} - \sum_{i=1}^{3} \left(\frac{1}{(\vec{k}_i - z\vec{p}_{2\pi})^2 + \mu^2} - \frac{1}{(\vec{k}_i - \bar{z}\vec{p}_{2\pi})^2 + \mu^2} \right). \tag{18}$$

In the case of a transversely polarized photon we introduce the transverse photon polarization vectors $\vec{\epsilon}(T=+,-)$ through

$$\vec{\epsilon}(+) = -\frac{1}{\sqrt{2}}(1,\mathrm{i}), \quad \vec{\epsilon}(-) = \frac{1}{\sqrt{2}}(1,-\mathrm{i}). \tag{19}$$

Using this, the upper impact factor for the Pomeron induced process can be written as

$$J_P^{\gamma_T^*}(\vec{k}_1,\vec{k}_2) = -\frac{\mathrm{i}eg^2\delta^{ab}}{4N_C}\int_0^1 \mathrm{d}z(z-\bar{z})\vec{\epsilon}(T)\cdot\vec{Q}_P(\vec{k}_1,\vec{k}_2) \times \Phi^{I=1}(z,\zeta,m_{2\pi}^2), \tag{20}$$

where the vector $\vec{Q}_P(\vec{k}_1,\vec{k}_2)$ is defined by

$$\vec{Q}_P(\vec{k}_1,\vec{k}_2) = \frac{z\vec{p}_{2\pi}}{z^2\vec{p}_{2\pi}^{\,2}+\mu^2} - \frac{\bar{z}\vec{p}_{2\pi}}{\bar{z}^2\vec{p}_{2\pi}^{\,2}+\mu^2} + \frac{\vec{k}_1 - z\vec{p}_{2\pi}}{(\vec{k}_1 - z\vec{p}_{2\pi})^2+\mu^2} - \frac{\vec{k}_1 - \bar{z}\vec{p}_{2\pi}}{(k_1 - \bar{z}\vec{p}_{2\pi})^2+\mu^2}. \tag{21}$$

The calculation of the Odderon exchange contribution gives

$$J_O^{\gamma_T^*}(\vec{k}_1,\vec{k}_2,\vec{k}_3) = -\frac{\mathrm{i}eg^3d^{abc}}{8N_C}\int_0^1 \mathrm{d}z(z-\bar{z}) \times \vec{\epsilon}(T)\cdot\vec{Q}_O(\vec{k}_1,\vec{k}_2,\vec{k}_3)\frac{1}{3}\Phi^{I=0}(z,\zeta,m_{2\pi}^2), \tag{22}$$

where we have used the definition

$$\vec{Q}_O(\vec{k}_1,\vec{k}_2,\vec{k}_3) = \frac{z\vec{p}_{2\pi}}{z^2\vec{p}_{2\pi}^{\,2}+\mu^2} + \frac{\bar{z}\vec{p}_{2\pi}}{\bar{z}^{\,2}\vec{p}_{2\pi}^{\,2}+\mu^2} + \sum_{i=1}^{3}\left(\frac{\vec{k}_i - z\vec{p}_{2\pi}}{(\vec{k}_i - z\vec{p}_{2\pi})^2+\mu^2} + \frac{\vec{k}_i - \bar{z}\vec{p}_{2\pi}}{(k_i - \bar{z}\vec{p}_{2\pi})^2+\mu^2}\right). \tag{23}$$

The value of the strong coupling constant g in the hard block is assumed to correspond to the 1-loop running coupling constant with $n_f = 2$,

$$\alpha_\mathrm{s}(Q^2) = \frac{g^2}{4\pi} = 12\pi \Bigg/ \left[29\ln\left(\frac{Q^2}{\Lambda_\mathrm{QCD}^2}\right)\right].$$

In our numerical estimates we take as a mean value $\Lambda_\mathrm{QCD} = 0.25\,\mathrm{GeV}$. Varying this value in a reasonable range does not modify much our results.

3.2 Generalized two pion distribution amplitudes

A crucial point of the present study is the choice of an appropriate two pion distribution amplitude (GDA) [17, 22,23] which includes the full strong interaction related to the production of the two pion system. We follow here the discussion in our previous paper [16] and propose a possible improvement of this GDA in Sect. 5.

The Odderon induced contribution we are looking for is directly proportional to the $I=0$ part of the GDA, for which we use the following approximation:

$$\Phi^{I=0}(z,\zeta,m_{2\pi}) = 10z\bar{z}(z-\bar{z})R_\pi \times \Big[-\frac{3-\beta^2}{2}\mathrm{e}^{\mathrm{i}\delta_0(m_{2\pi})}|\mathrm{BW}_{f_0}(m_{2\pi}^2)| + \beta^2\mathrm{e}^{\mathrm{i}\delta_2(m_{2\pi})}|\mathrm{BW}_{f_2}(m_{2\pi}^2)|P_2(\cos\theta)\Big], \tag{24}$$

with $R_\pi = 0.5$ and β given by (11).

In our studies we fix the shapes of the phase shifts δ_0 and δ_2 by a fit to the data presented in [26]. The factors $|\mathrm{BW}_{f_{0,2}}(m_{2\pi}^2)|$ are the modulus of the Breit–Wigner amplitudes

$$\mathrm{BW}_{f_0}(m_{2\pi}^2) = \frac{m_{f_0}^2}{m_{f_0}^2 - m_{2\pi}^2 - im_{f_0}\Gamma_{f_0}}, \quad m_{f_0} = 0.98\,\mathrm{GeV}, \quad \Gamma_{f_0} = 0.075\,\mathrm{GeV}, \tag{25}$$

$$\mathrm{BW}_{f_2}(m_{2\pi}^2) = \frac{m_{f_2}^2}{m_{f_2}^2 - m_{2\pi}^2 - im_{f_2}\Gamma_{f_2}}, \quad m_{f_2} = 1.275\,\mathrm{GeV}, \quad \Gamma_{f_2} = 0.186\,\mathrm{GeV}. \tag{26}$$

Modifying the f_0 width changes slightly our results, as discussed in [16].

For the isospin $I=1$ part of the two pion GDA, which is relevant for the Pomeron exchange amplitude, we take

$$\Phi^{I=1}(z,\zeta,m_{2\pi}) = 6z\bar{z}\beta\cos\theta F_\pi(m_{2\pi}^2), \tag{27}$$

where the time-like pion form factor is parameterized by

$$F_\pi(m_{2\pi}^2) = \frac{1}{(1-0.145)}\mathrm{BW}_\rho\frac{1+1.85\cdot 10^{-3}\cdot\mathrm{BW}_\omega}{1+1.85\cdot 10^{-3}}, \tag{28}$$

with

$$\mathrm{BW}_\rho(m_{2\pi}^2) = \frac{m_\rho^2}{m_\rho^2 - m_{2\pi}^2 - \mathrm{i}\sqrt{m_{2\pi}^2}\,\Gamma_\rho(m_{2\pi}^2)}, \quad \Gamma_\rho(m_{2\pi}^2) = \Gamma_\rho\frac{m_\rho^2}{m_{2\pi}^2}\frac{(m_{2\pi}^2-4m_\pi^2)^{3/2}}{(m_\rho^2-4m_\pi^2)^{3/2}}, \quad m_\rho = 0.773\,\mathrm{GeV}, \quad \Gamma_\rho = 0.145\,\mathrm{GeV} \tag{29}$$

and

$$\mathrm{BW}_\omega(m_{2\pi}^2) = \frac{m_\omega^2}{m_\omega^2 - m_{2\pi}^2 - \mathrm{i}m_\omega\Gamma_\omega}, \quad m_\omega = 0.782\,\mathrm{GeV}, \quad \Gamma_\omega = 0.0085\,\mathrm{GeV}. \tag{30}$$

The phase of the form factor $F_\pi(m_{2\pi}^2)$ will be denoted by $\mathrm{e}^{\mathrm{i}\delta_1}$, where δ_1 is the corresponding p wave phase shift.

3.3 Proton impact factors

Finally we have to fix the lower soft parts of our amplitudes, i.e. the proton impact factors. They cannot be calculated within perturbation theory. In our estimates we will use phenomenological eikonal models of these impact factors proposed in [24] and [25]. We take for the Pomeron exchange

$$J_P^{N\to N'} = \mathrm{i}\frac{\bar{g}^2\delta^{ab}}{2N_C}3\left[\frac{A^2}{A^2+\frac{1}{2}\vec{p}_{2\pi}^{\,2}} - \frac{A^2}{A^2+\frac{1}{2}(\vec{k}_1^2+\vec{k}_2^2)}\right], \tag{31}$$

and for the Odderon exchange

$$J_O^{N\to N'} = -\mathrm{i}\frac{\bar{g}^3 d^{abc}}{4N_C}3\left[F(\vec{p}_{2\pi},0,0) - \sum_{i=1}^{3}F(\vec{k}_i,\vec{p}_{2\pi}-\vec{k}_i,0) + 2F(\vec{k}_1,\vec{k}_2,\vec{k}_3)\right], \tag{32}$$

where

$$F(\vec{k}_1,\vec{k}_2,\vec{k}_3) = \frac{A^2}{A^2+\frac{1}{2}\left[(\vec{k}_1-\vec{k}_2)^2+(\vec{k}_2-\vec{k}_3)^2+(\vec{k}_3-\vec{k}_1)^2\right]} \tag{33}$$

and $A = m_\rho/2$. In these equations we have denoted the soft QCD-coupling constant by $\bar{g}$. We take $\alpha_{\mathrm{soft}} = \bar{g}^2/(4\pi) = 0.5$ as a reasonable mean value (for discussion of this point see our paper [16]).

4 Asymmetries and their numerical evaluation

Taking together (13)–(15) and (17), (20), (22), (24), (27), (31) and (32) we are now ready for the calculation of the asymmetries and their subsequent numerical evaluation. In contrast to the results presented in [16] where we only considered the charge asymmetry resulting from the scattering of a longitudinally polarized photon, we consider below the contributions to the charge asymmetry comming from both longitudinal and transverse photon degrees of freedom. Moreover, we study the single-spin asymmetry which involves amplitudes with transversely and longitudinally polarized photons.

Because all photon polarizations contribute to the asymmetries they will now depend on the angle α between the initial electron transverse momentum $\vec{p}_i$ and the tranverse momentum of the pion pair $\vec{p}_{2\pi}$, and on the energy loss y (see (5)) of the initial electron.

4.1 Charge asymmetry

We define the forward–backward or charge asymmetry by

$$A(Q^2,t,m_{2\pi}^2,y,\alpha) = \frac{\sum_{\lambda=+,-}\int\cos\theta\,\mathrm{d}\sigma(s,Q^2,t,m_{2\pi}^2,y,\alpha,\theta,\lambda)}{\sum_{\lambda=+,-}\int\mathrm{d}\sigma(s,Q^2,t,m_{2\pi}^2,y,\alpha,\theta,\lambda)} = \frac{\int\mathrm{d}\cos\theta\cos\theta N_{\mathrm{charge}}}{\int\mathrm{d}\cos\theta D}. \tag{34}$$

We observe that the vectors $\vec{Q}_{P/O}$ in (21, 23), after integration over the gluon momenta $\vec{k}_i$, can be only proportional to $\vec{p}_{2\pi}$. Therefore it is useful to define scalar functions $\mathcal{A}_{\mathrm{T}}(P/O)$ by

$$\vec{\epsilon}(T)\cdot\vec{p}_{2\pi}\mathcal{A}_{\mathrm{T}}(P/O) \equiv \vec{\epsilon}(T)\cdot\vec{Q}_{P/O}. \tag{35}$$

Using this, the calculation of the numerator N_{charge} and the denominator D gives

$$\begin{aligned} N_{\mathrm{charge}} &= 8(1-y)\mathrm{Re}\left[\mathcal{M}_{\mathrm{L}}(P)\mathcal{M}_{\mathrm{L}}^*(O)\right] \\ &+ 4(2-y)\sqrt{1-y}|\vec{p}_{2\pi}|\cos\alpha \\ &\times \mathrm{Re}\left[\mathcal{A}_{\mathrm{T}}(P)\mathcal{M}_{\mathrm{L}}^*(O)+\mathcal{A}_{\mathrm{T}}(O)\mathcal{M}_{\mathrm{L}}^*(P)\right] \\ &+ 2(1+(1-y)^2+2(1-y)\cos 2\alpha)|\vec{p}_{2\pi}|^2 \\ &\times \mathrm{Re}\left[\mathcal{A}_{\mathrm{T}}(P)\mathcal{A}_{\mathrm{T}}^*(O)\right] \end{aligned} \tag{36}$$

and

$$\begin{aligned} D &= 4(1-y)\left|\mathcal{M}_{\mathrm{L}}(P)+\mathcal{M}_{\mathrm{L}}(O)\right|^2 \\ &+ 4(2-y)\sqrt{1-y}|\vec{p}_{2\pi}|\cos\alpha \\ &\times \mathrm{Re}\left[(\mathcal{A}_{\mathrm{T}}(P)+\mathcal{A}_{\mathrm{T}}(O))\left(\mathcal{M}_{\mathrm{L}}^*(P)+\mathcal{M}_{\mathrm{L}}^*(O)\right)\right] \\ &+ (1+(1-y)^2+2(1-y)\cos 2\alpha)|\vec{p}_{2\pi}|^2 \\ &\times \left|\mathcal{A}_{\mathrm{T}}(P)+\mathcal{A}_{\mathrm{T}}(O)\right|^2. \end{aligned} \tag{37}$$

Instead of a weighted integration of the cross section over θ it is possible to perform a full angular analysis. The numerator of the asymmetry would then be provided by the $\cos\theta$ term which is characteristic for the longitudinal polarization of the pion pair.

We checked that the squared Odderon contribution in the denominator can be neglected (except for the large y region), so that the asymmetry is practically a measure of the ratio of the Odderon and the Pomeron amplitudes.

Before presenting our results for the asymmetries and in order to get some handle on relative counting rates we show in Fig. 3 the α and y behavior of the denominator D (in arbitrary units), which is directly proportional to the charge averaged differential cross section. One sees clearly that the denominator is only about a factor of 2–4 below its maximum value at $y = 0$ and $\alpha = \pi/2$ in a region which is experimentally accessible and where the asymmetry to be discussed below is relatively large.

The characterisic $m_{2\pi}$ dependence shown in Fig. 4 is completely understood in terms of the $\pi\pi$ phase shifts and the factor $\sin(\delta_{0,2}-\delta_1)$. The phase difference vanishes for $m_{2\pi} \approx 0.75$ GeV and $m_{2\pi} \approx 1$ GeV resulting in two zeros of the charge asymmetry. The magnitude of the charge asymmetry is quite large around the f_0 and f_2 masses. It depends somewhat on the width of the f_0-meson which is taken to be 0.075 GeV (see the discussion in [16]). The Q^2 dependence of the charge asymmetry is, as seen from Fig. 4, moderate but of course the cross section increases with decreasing Q^2.

The α and y dependences of the charge asymmetry are shown in Figs. 5 and 6 for a value of $m_{2\pi}$ just below the f_0 mass, and on Figs. 7 and 8 for a value of $m_{2\pi}$ just equal to the f_2 mass, where the asymmetry is large. The effect is maximal for values of $\alpha \approx 0$ and minimal for $\alpha \sim \pi$. The

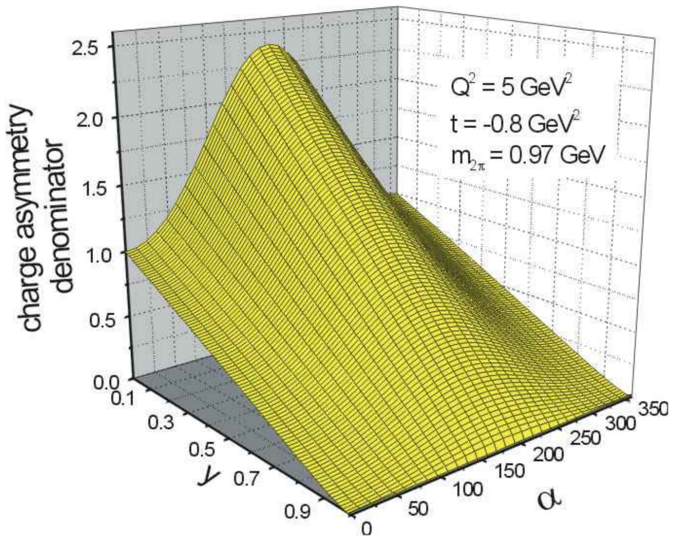

Fig. 3. (y, α) dependence of the denominator (37) of the asymmetries for $m_{2\pi} = 0.97\,\mathrm{GeV}$, $t = -0.8\,\mathrm{GeV}^2$ and $Q^2 = 5\,\mathrm{GeV}^2$

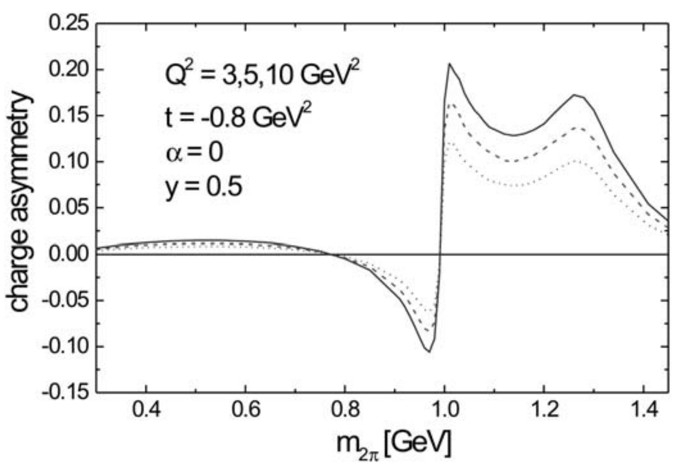

Fig. 4. $m_{2\pi}$ dependence of the charge asymmetry for $Q^2 = 3$ GeV^2 (*solid line*), 5 GeV^2 (*dashed line*) and 10 GeV^2 (*dotted line*) for $t = -0.8\mathrm{GeV}^2$, $\alpha = 0$ and $y = 0.5$

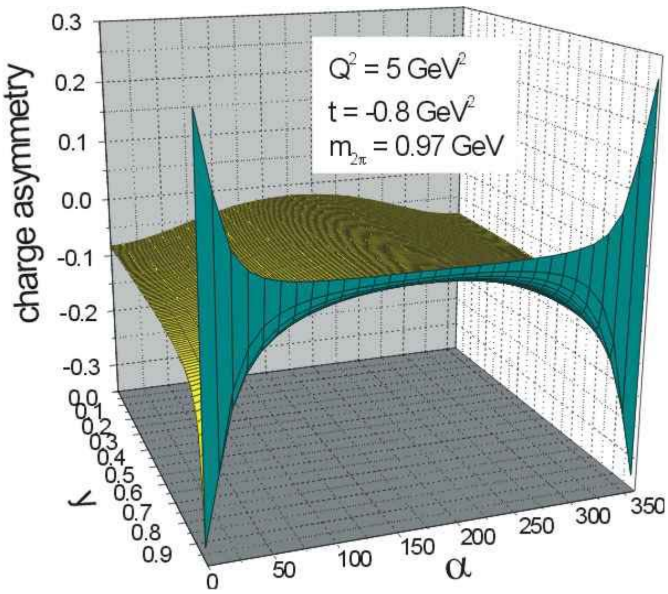

Fig. 5. (y, α) dependence of the charge asymmetry for $m_{2\pi} = 0.97\,\mathrm{GeV}$, $t = -0.8\,\mathrm{GeV}^2$ and $Q^2 = 5\,\mathrm{GeV}^2$ seen from the α side

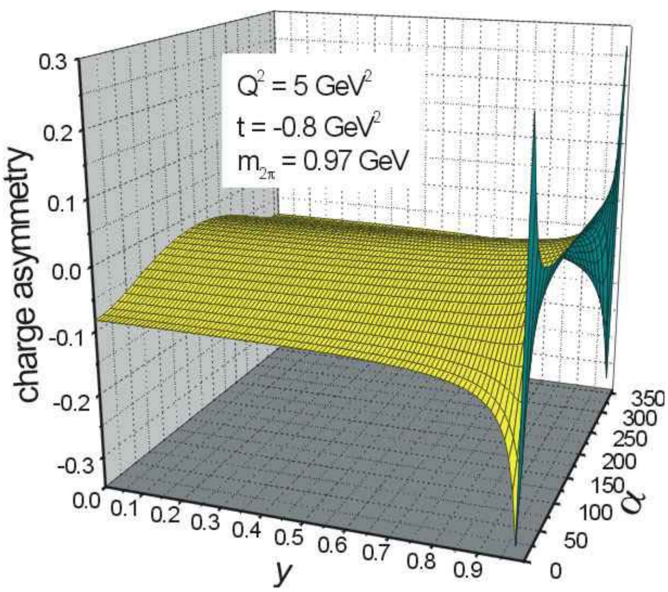

Fig. 6. (y, α) dependence of the charge asymmetry for $m_{2\pi} = 0.97\,\mathrm{GeV}$, $t = -0.8\,\mathrm{GeV}^2$ and $Q^2 = 5\,\mathrm{GeV}^2$ seen from the y side

dependence on y is very weak except for the region $y \to 1$ where the cross section is so small that no experimental data will ever be available.

The t dependence of the asymmetry is plotted in Fig. 9. It has a characteristic zero around $t = -0.06\,\mathrm{GeV}^2$. This zero in the Odderon amplitude has already been discussed in [13].

In Fig. 10 we show an error band for the $m_{2\pi}$-dependent charge asymmetry resulting from a simultaneous variation of Λ_{QCD} and the soft coupling α_{soft} in the indicated range.

4.2 Spin asymmetry

The presence of the interference between the different helicity amplitudes with non-zero phase shift between them provides the necessary conditions for the emergence of single spin asymmetries.

The most realistic is the single-spin asymmetry generated by the polarized lepton beam, which is accessible at HERA. The resulting azimuthal asymmetry is analogous to the ones measured at lower energies by the HERMES and CLAS collaborations [20].

The single spin asymmetry is defined by

$$A_S(Q^2, t, m_{2\pi}^2, y, \alpha) = \frac{\sum\limits_{\lambda=+,-} \lambda \int \cos\theta \, \mathrm{d}\sigma(s, Q^2, t, m_{2\pi}^2, y, \alpha, \theta, \lambda)}{\sum\limits_{\lambda=+,-} \int \mathrm{d}\sigma(s, Q^2, t, m_{2\pi}^2, y, \alpha, \theta, \lambda)}$$

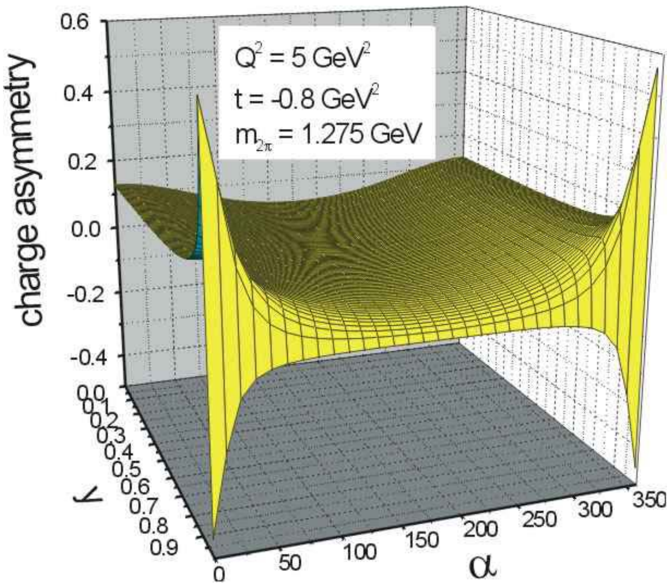

Fig. 7. (y, α) dependence of the charge asymmetry for $m_{2\pi} = 1.275\,\mathrm{GeV}$, $t = -0.8\,\mathrm{GeV}^2$ and $Q^2 = 5\,\mathrm{GeV}^2$ seen from the α side

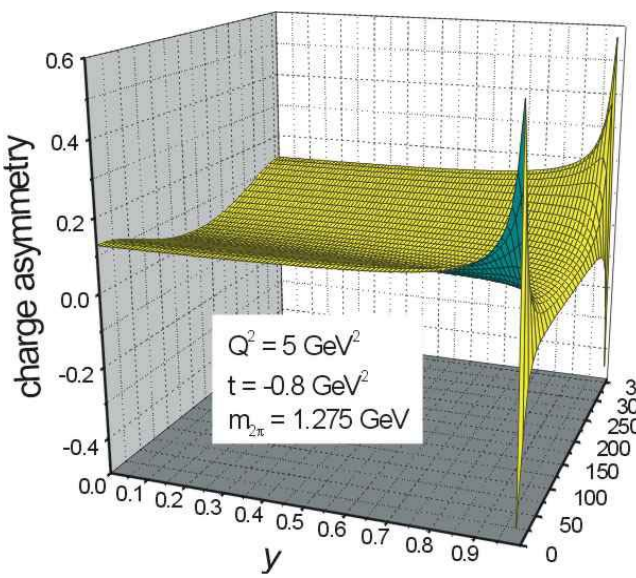

Fig. 8. (y, α) dependence of the charge asymmetry for $m_{2\pi} = 1.275\,\mathrm{GeV}$, $t = -0.8\,\mathrm{GeV}^2$ and $Q^2 = 5\,\mathrm{GeV}^2$ seen from the y side

$$= \frac{\int \mathrm{d}\cos\theta \cos\theta \; N_{\mathrm{spin}}}{\int \mathrm{d}\cos\theta D}, \tag{38}$$

and the calculation of the numerator gives

$$N_{\mathrm{spin}} = 4y\sqrt{1-y}\sin\alpha |\vec{p}_{2\pi}| \times \mathrm{Im}\left[\mathcal{M}_{\mathrm{L}}(P)\mathcal{A}^*_{\mathrm{T}}(O) + \mathcal{M}_{\mathrm{L}}(O)\mathcal{A}^*_{\mathrm{T}}(P)\right], \tag{39}$$

while D is of course the same quantity as in the case of the charge asymmetry (37). In order to increase the magnitude of the spin asymmetry we defined it, by analogy to the charge asymmetry (34), with an integration over the angle θ (or the variable ζ, see (11)) weighted with $\cos\theta$. The integration over θ without a weight factor gives in our approach zero.

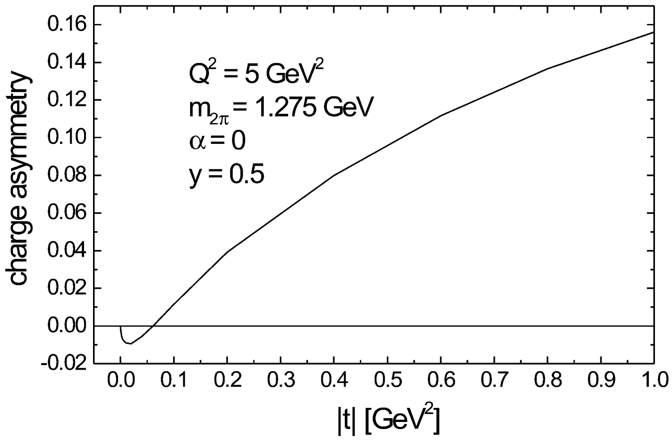

Fig. 9. t dependence of the charge asymmetry at $m_{2\pi} = 1.275\,\mathrm{GeV}$, $Q^2 = 5\,\mathrm{GeV}^2$

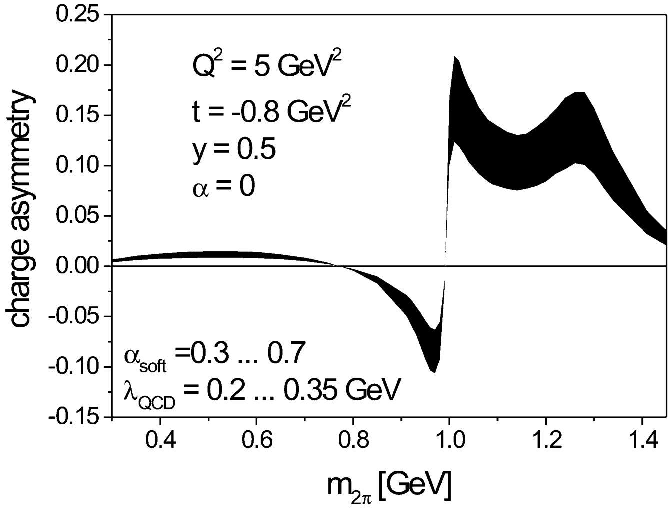

Fig. 10. Error bands resulting from a variation of Λ_{QCD} and the soft coupling α_{soft}

Here again the asymmetry (38) measures the interference between the Pomeron and Odderon exchange amplitudes. As is obvious from this equation, the effect is maximal for α near $\pi/2$. The dependence on $m_{2\pi}$ is shown in Fig. 11 for different values of Q^2. The $m_{2\pi}$ dependence is quite complementary to the case of charge asymmetry since an additional factor of i comes from the helicity difference in the leptonic trace, so that the strong phase accumulates an additional factor of $\pi/2$.

Indeed, as the Pomeron amplitude is imaginary and the Odderon one is real the relative phase between them is the maximal one for the emergence of single-spin asymmetries [21]. The effect should therefore be maximal for a zero relative phase between the isoscalar and isovector distributions, providing a complementary probe. Therefore, simultaneous studies of charge and spin asymmetries provide an important cross-check.

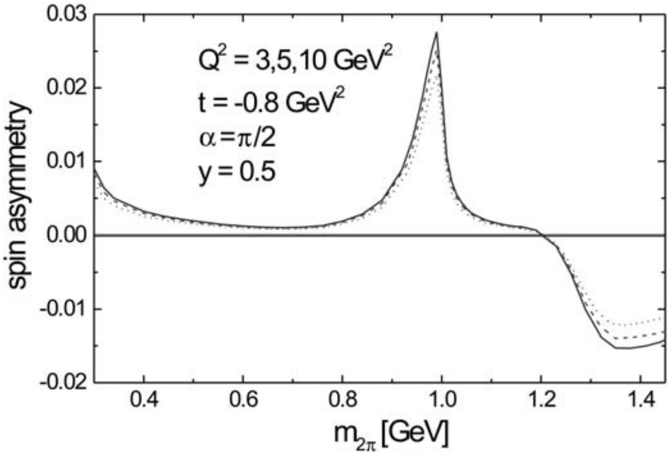

Fig. 11. $m_{2\pi}$ dependence of the spin asymmetry at $t = -0.8\,\mathrm{GeV}^2$, $Q^2 = 3$ (*solid line*), 5 (*dashed line*), 10 (*dotted line*) GeV^2

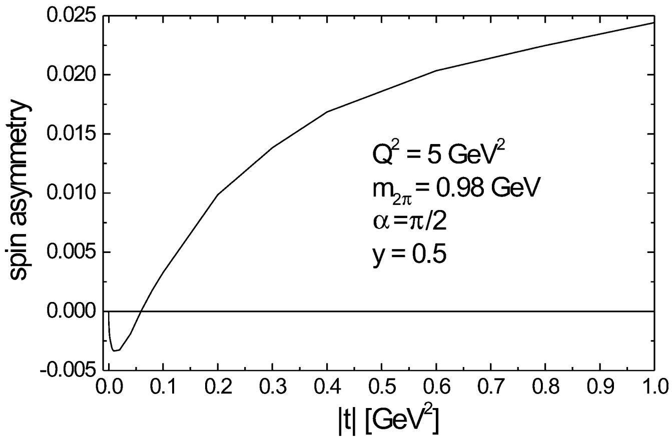

Fig. 12. t dependence of the spin asymmetry at $m_{2\pi} = 0.98\,\mathrm{GeV}^2$, $Q^2 = 5\,\mathrm{GeV}^2$

The resulting $m_{2\pi}$ dependence has thus a characteristic $\cos(\delta_{0,2} - \delta_1)$ shape, modulated by the absolute values of the ρ, f_0 and f_2 Breit–Wigner amplitudes. The spin asymmetry is maximal when $(\delta_{0,2} - \delta_1)$ is equal to 0 or π, i.e. around 0.98 and 1.32 GeV. Let us also note that the Q^2 dependence of the spin asymmetry (see Fig. 11) is much weaker than in the case of charge asymmetry; see Fig. 4. Unfortunately the magnitude of the spin asymmetry is quite small (although comparable to the recent measurement [20]) at low t, where the charge asymmetry is sizable.

The t dependence of the spin asymmetry is shown in Fig. 12.

5 Sensitivity to the GDA

The characteristic $m_{2\pi}$ dependence of the asymmetries comes entirely from the choice of the two pion distribution amplitude. As we already stressed, this GDA is a non-perturbative object which we cannot claim to know at present. Our model was mostly guided by an optimistic expansion of the range of validity in $m_{2\pi}$ of the Watson theorem up to 1.0 and even 1.5 GeV. Other models [22] did not use such an assumption, with the important consequences that neither the drastic phase shift increase near the f_0 mass [26], nor the magnitude peak related to it, do appear in the GDA and therefore in the asymmetries. Because of that we expect that after taking into account the f_0-resonance the estimates of [22] around 1 GeV (where $f_0(980)$ contributes) will be modified. On the other hand, treating the $\pi\pi$ interaction near 1 GeV without mentioning the problem of inelasticity and the opening of the $K\bar{K}$ threshold is likely to be unrealistic too. In order to get an estimate of the effects which may arise due to the opening of the $K\bar{K}$ threshold, we implemented a modified GDA in our calculation of the charge asymmetry, which differs from the one given in (24) by the inclusion of an inelasticity factor $\eta(m_{2\pi})$ in front of the f_0-resonance in (24) as obtained in the analysis of [30]. We show in Fig. 13 the charge asymmetry obtained with such a modified GDA at $Q^2 = 5\,\mathrm{GeV}^2$, $t = -0.8\,\mathrm{GeV}^2$ and $\alpha = 0$. The effect of this new parametrization is obviously a decrease of the charge asymmetry above the $K\bar{K}$ threshold. This change of GDA also moderately modifies the charge asymmetry in the vicinity of the f_2-resonance.

One may also adopt an alternative point of view and take these experiments as another way (together with $\gamma^*\gamma$ reactions [23]) to determine the two pion distribution amplitude, once the dependence of the asymmetries on variables such as s, Q^2, t and α has been checked.

6 Remarks on possible effects of QCD evolution

The most natural improvement of our results, specially for the charge asymmetry, consists in the inclusion of the BFKL and BKP evolution in the scattering amplitudes with Pomeron and Odderon exchanges, $\mathcal{M}_P$ and $\mathcal{M}_O$, respectively. This is beyond the scope of the present paper, but nevertheless we can draw some qualitative conclusions about their possible effects. There is no s dependence at the Born level, provided s is large enough for the usual high energy approximation to hold. The BFKL and BKP evolutions introduce characteristic energy dependences. They can also lead to some changes of the normalization of the involved amplitudes, as well as to the appearance of some additional phases δ_P and δ_O.

The charge asymmetry is effectively the product of

$$\frac{|\mathcal{M}_O|}{|\mathcal{M}_P|} \cdot \sin(\delta_{0,2} - \delta_1).$$

We believe that the inclusion of BFKL and BKP evolution does not change dramatically the ratio $|\mathcal{M}_O|/|\mathcal{M}_P|$. On the other hand, the possible appearance of an additional phase difference $\delta = \delta_P - \delta_O$ would lead to a change of the argument of the sine above. Let us however note that the structure of the charge asymmetry with two zeros in

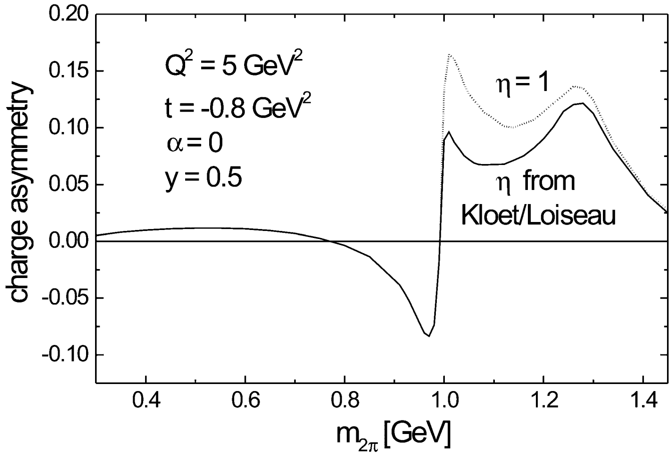

Fig. 13. $m_{2\pi}$ dependence of the charge asymmetry with the inelasticity factor $\eta(m_{2\pi})$ (*solid curve*) and with $\eta(m_{2\pi}) = 1$ (*dotted curve*) for $Q^2 = 5\,\mathrm{GeV}^2$, $t = -0.8\,\mathrm{GeV}^2$, $\alpha = 0$, $y = 0.5$

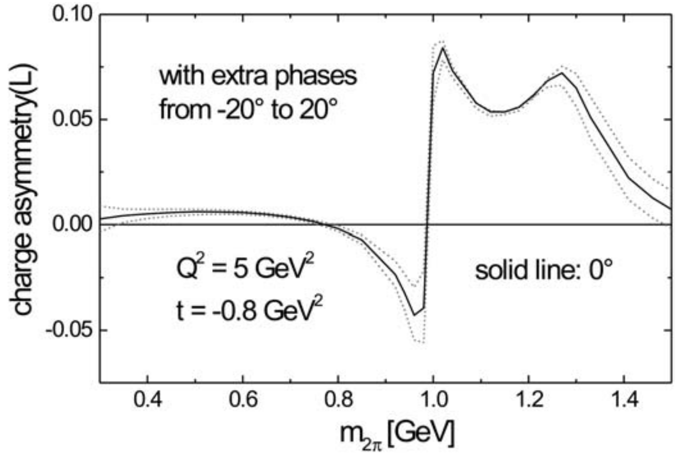

Fig. 14. $m_{2\pi}$ dependence of the charge asymmetry from the longitudinal photon with an additional constant phase δ

Fig. 4 is robust against a moderate (independent of $m_{2\pi}$) phase δ. The rapid change of the δ_0 phase shift near $m_{2\pi} = 1\,\mathrm{GeV}$ (see [26]) enforces a zero of the asymmetry, even if an additional "extra" phase is introduced. The same is true for the zero at $m_{2\pi} \approx 0.8\,\mathrm{GeV}$. There in contrast to the upper argumentation the rapid change of the pion form factor phase shift δ_1 enforces the zero.

As an illustration of these remarks we present in Fig. 14 the longitudinal charge asymmetry at $Q^2 = 5\,\mathrm{GeV}^2$ and $t = -0.8\,\mathrm{GeV}^2$, calculated for two values of the additional phase $\delta = \pm 20°$ (dashed lines). They resulting curves differ very little from the original curve corresponding to $\delta = 0°$ (solid line).

7 Remarks on HERMES data

Our discussion here is restricted to diffractive physics, mostly testable in collider experiments, where center of mass energies are of the order of 100 GeV and more. At lower energies, exclusive electroproduction of pairs of mesons are described in the framework of the collinear factorization, the soft part of the amplitude being represented by generalized parton distributions [28]. In such a framework quark–antiquark and two-gluon exchanges play the dominant role. Here a charge asymmetry may occur as the result of the interference of charge parity odd and charge parity even amplitudes. An estimate of this asymmetry has been computed in [27], using an isosinglet generalized distribution amplitude which differs from our choice in that it does not include the f_0-resonance[1]. Recent data from the HERMES experiment [29] at HERA are indeed compatible with these estimates and show confusingly no sign of the f_0-resonance.

The estimates in [27] lead to a very small value of the asymmetries at low x_{Bj}. This is easily understandable since in this region gluon exchange diagrams dominate which select charge parity odd mesonic states, leading to vanishing interference effects. This opens the interesting possibility that the data unravel an interference effect between two- and three-gluon exchange, which would in that framework be understood as a higher twist contribution. In such a fixed target experiment, a small x_{Bj} value is indeed related to quite low values of Q^2 and therefore to higher twist contributions. One may also understand such an effect as an early sign of Pomeron–Odderon interference. In any case, pushing the analysis to the lowest possible x_{Bj} values is extremely interesting. No single lepton spin asymmetries should show up in these lower energy data if the leading twist contribution is indeed dominant. The reason is the asymptotic dominance of the process with a longitudinally polarized virtual photon. Here also higher twist contributions may yield sizable spin asymmetries at quite low values of Q^2.

8 Conclusion

Our study shows that the role of the Odderon in diffractive processes in perturbative QCD is intimately related to a sizable charge asymmetry in the electroproduction of two charged mesons. The single-spin asymmetry in the same reaction turned out to be much smaller. We applied the powerful tool of QCD factorization which allows us to calculate the hard subprocess perturbatively, while the soft ingredients (GDA and proton impact factor) should be modeled or, better, measured, which poses a new challenging problem for experimentalists.

Let us finally emphasize that data on this reaction in the kinematical domain suitable for our calculation (i.e. large s, small t, Q^2 above $1\,\mathrm{GeV}^2$ and $m_{2\pi}$ below 1.5 GeV) should be easily accessible for analysis by the experimental set-ups H1 [9] and ZEUS [31] at HERA. Such a confrontation of theory with experiment should shed some light on the status of the Odderon.

[1] We thank B. Lehmann-Dronke and M. Polyakov for discussions on this point

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